$\mathscr{E}'$ as an algebra by multiplicative convolution and Hadamard type operators on smooth functions

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We study the algebra $\mathscr{E}'(\mathbb{R}^d)$ equipped with the multiplication $(T \star S)(\varphi) = T_x(S_y(f(xy))$ where $xy = (x_1y_1, \ldots, x_dy_d)$. For open sets Ω', $\Omega \subset \mathbb{R}^d$ we determine the distributions $T \in \mathscr{E}'(\mathbb{R}^d)$ such that $T \star \mathscr{E}'(\Omega') \subset \mathscr{E}'(\Omega)$, in particular, the T which operate on $\mathscr{E}'(\Omega)$. They are distributions whose support is contained in the dilation sets $V(\Omega', \Omega)$ or $V(\Omega)$, respectively. By transposition we obtain a characterization of the $\star$-convolution operators which send $C^\infty(\Omega)$ to $C^\infty(\Omega')$ and, in particular, the $\star$-convolution operators on $C^\infty(\Omega)$. These are called Hadamard-type operators and they are characterized by the property that they admit all monomials as eigenvectors. The algebra $M(\Omega)$ of such operators is a closed subalgebra of $L(C^\infty(\Omega))$ and we determine the topology induced from $L_b(C^\infty(\Omega))$ on $\mathscr{E}'(V(\Omega))$. We show that the algebra $M(\Omega)$ is isomorphic to an algebra of holomorphic functions around zero where multiplication is the classical Hadamard multiplication, that is, multiplication of the Taylor coefficients. Hadamard-type operators assigned to distributions with support $\{(1, \ldots, 1)\}$ are called Euler operators and we study global solvability for such operators on open subsets of $\mathbb{R}^d_+$.

Analogous problems for real analytic functions are studied in papers of Domański-Langenbruch and a paper of Domański-Langenbruch-Vogt. While the problems are analogous, the results, the methods and the difficulties to overcome are, in part, quite different.