Growth bound and spectral bound for semigroups on Fréchet spaces

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Let X be a Fréchet space. A family of operators $(T(t))_{t\geq 0} \subseteq L(X)$ is a C_0-semigroup if it satisfies the evolution property $T(t)T(s) = T(t+s)$ for s, $t \geq 0$ and $T(0) = \mathrm{id}_X$, and if all its orbits are continuous. Its generator $A\colon D(A) \to X$ is defined as the differential of the orbit $t \mapsto T(t)x$ at $t = 0$ on those $x \in X$ for which this limit exists.

In the talk, we define the *growth bound* of T as the infimum over those real ω for which $\{e^{-\omega t}T(t)\,;\, t \geq 0\} \subseteq L(X)$ is equicontinuous. Then we use classical and recent approaches for non Banach spectral theories to define the spectral bound $\mathrm{s}(A)$ as the supremum over the realparts of all complex points of the spectrum of A. We study the two bounds and show that the Banach space inequality $\mathrm{s}(A) \leq \omega_0(T)$ extends to Fréchet spaces. In addition, we discuss several concrete examples.