Bases in spaces of analytic functions and applications

Vyacheslav Zakharyuta

Sabancı University, Turkey
zaha@sabanciuniv.edu

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Let Ω be a Stein manifold. If D is an open subset of Ω, we denote by $A(D)$ the locally convex space of all functions analytic in D with the topology of locally uniform convergence in D. For an arbitrary set $E \subset \Omega$, $A(E)$ is a set of all analytic germs on E considered with the locally convex topology of the inductive limit:

$$A(E) = \text{limind}_{G \in \mathcal{O}(E)} A(G),$$

where $\mathcal{O}(E)$ is the set of all open neighborhoods of E.

Our goal is to give a survey of results on Schauder bases in those spaces (especially, for the cases when E is an open or compact set in Ω). Main attention will be paid to the following topics: existence of bases in spaces $A(E)$, their construction and structure, extendible bases; orthogonal, doubly orthogonal and interpolation bases, application to isomorphic classification of spaces of analytic functions; applications to approximation, interpolation and extension of analytic functions. The one-dimensional case is considered separately, since some results can be proved easier than their multivariate counterparts, the others have no many-dimensional analogs at all. This is due to some specific one-dimensional tools (like the fundamental algebra theorem, or the Grothendieck-Köthe-Silva duality, or the potential theory in $\mathbb{R}^2 = \mathbb{C}$). For $\dim \Omega \geq 2$ the pluripotential theory and functional analysis methods play main role. Some long standing open problems will be discussed.