

Generalized Brownian processes, random matrices, α determinant and positive definite functions on Coxeter (permutation) groups

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In my talk we will consider the following subjects:

1. Positive definite functions on Coxeter groups (W,S) connected with (a) the length function $|x| = \min\{k : x = s_1 \dots s_k, s_j \in S, \text{ and representation is minimal}\}$ and we show that the function $P(x, q) = q^{|x|}$ is positive definite on W for q in interval $[-1, 1]$, (b) also for the "block" length function $\|x\| =$ the number of different generators in the representation of x as above, we prove that the function $Q(x, t) = t^{\|x\|}$ is positive definite on W for t in interval $[0, 1]$.
2. Applications: (a) The construction of q -CCR relations $A(f)A^*(g) - qA^*(g)A(f) = \langle f, g \rangle$ for f, g in a Hilbert space, for real q and complex $|q| = 1$, and connections with theta function of Jacobi, (b) Free infinite divisibility of classical normal law $N(0, 1)$, the distributions like $1/\cosh$ and others classical Meixner laws.
3. Markov random matrices and special positive definite functions on infinite permutation group related to the number of "isolated" fixed points of a permutation.
4. Applications to positivity results for q -determinants and α -determinants.

References

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