

The Hardy space H^1 in the rational Dunkl setting

Jacek Dziubański

Uniwersytet Wrocławski, Poland
jdzuban@emath.uni.wroc.pl

The talk is based mostly on a joint work with Jean-Philippe Anker (Université d'Orléans). A part of this work was done in collaboration with Néjib Ben Salem (Faculté des Sciences de Tunis), and Nabila Hamda (Faculté des Sciences de Tunis)

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In the Euclidean setting, the classical real Hardy space $H^1(\mathbf{R}^n)$ can be characterized in several ways, for instance by means of

- atoms,
- the heat maximal operator,
- the Poisson maximal operator,
- Riesz transforms.

We have extended these characterizations to the rational Dunkl setting, first in dimension one and next in the product case. The proofs are based on a precise behavior of the heat kernel for the Dunkl Laplacian. At this occasion we have observed that the heat kernel fails to satisfy Gaussian upper bounds. Its decay and smoothness is nevertheless sufficient in order to apply Uchiyama's approach for characterization of Hardy spaces on spaces of homogeneous type in the sense of Coifman-Weiss. The general case would require sharp heat kernel estimates, which are not available for the time being.

In the talk we shall present our results, almost from scratch, including a multiplier theorem of Hörmander type for the Dunkl transform on H^1 and its application.