

Convolution of orbital measures on symmetric spaces

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Session: 18. Harmonic analysis, orthogonal expansions and Dunkl theory

The presented results are based on a series of recent articles common with P. Sawyer (Laurentian University, Sudbury, Canada).

We consider a Riemannian symmetric space of non-compact type G/K and we study the existence of the density of the convolution

$$\delta_{e^X}^{\natural} \star \delta_{e^Y}^{\natural}$$

of two orbital measures, when X and Y are singular elements of the Cartan space. This density intervenes in the product formula for spherical functions. Studying of its existence is also motivated by applications in probability theory on G/K .

We will survey our earlier results in this area: the existence of the density when X is regular and $Y \neq 0$ and the characterization of singular X, Y such that the density exists for the spaces $\mathbf{SL}(n, \mathbf{F})/\mathbf{SU}(n, \mathbf{F})$, as well as their complex and quaternionic versions.

Our recent results concern the symmetric spaces of type B_p, C_p et D_p , i.e. the non-compact Grassmannians $\mathbf{SO}_0(p, q)/\mathbf{SO}(p) \times \mathbf{SO}(q)$ and the symmetric spaces $\mathbf{SO}_0(p, p)/\mathbf{SO}(p) \times \mathbf{SO}(p)$, $\mathbf{SU}(p, p)/\mathbf{S}(\mathbf{U}(p) \times \mathbf{U}(p))$ and $\mathbf{Sp}(p, p)/\mathbf{Sp}(p) \times \mathbf{Sp}(p)$.

We will finish by discussing analogous problems on symmetric spaces of Euclidean type and of compact type.

References

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