Hypoelliptic operators and sharp multiplier theorems

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Let L be the Laplacian on $\mathbb{R}^n$. The investigation of necessary and sufficient conditions for an operator of the form $F(L)$ to be bounded on L^p in terms of "smoothness properties" of the spectral multiplier F is a classical research area of harmonic analysis, with long-standing open problems (e.g., the Bochner-Riesz conjecture) and connections with the regularity theory of PDEs.

In settings other than the Euclidean, particularly in the presence of a sub-Riemannian geometric structure, the natural substitute L for the Laplacian need not be an elliptic operator, and it may be just hypoelliptic. In this context, even the simplest questions related to the L^p-boundedness of operators of the form $F(L)$ are far from being completely understood.

I will present some recent results, obtained in joint works with Detlef Müller (Kiel) and Adam Sikora (Sydney), dealing with the case of sublaplacians on 2-step stratified (Carnot) groups, and with Grushin operators.

References

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