Some results on a subspace of $L^2(\mathbb{R})$ spanned by shifts of a simple function

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Given a closed subspace $V \subset L^2(\mathbb{R})$ spanned by the integer shifts of a single function φ one considers the function $P_\varphi(\xi) = \sum_{k \in \mathbb{Z}} |\hat{\varphi}(\xi + 2\pi k)|^2$. It is known that the size properties of this function correspond to properties of the integer shifts of φ as the spanning set. We will discuss some of these correspondences. We will also present the recent solution (by Saliani), which uses a powerful theorem of Kislyakov, of a long standing conjecture (attributed to Weiss): $P_\varphi > 0$ a.e. if and only if the set $\{\varphi(\cdot - k) : k \in \mathbb{Z}\}$ is L^2-linearly independent.