Heat kernels on affine buildings

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Let $\mathscr{X}$ be a thick affine building of rank $r+1$. We consider a finite range isotropic random walk on vertices of $\mathscr{X}$. We show sharp lower and upper estimates on p_n, the n'th iteration of the transition operator, uniform in the region

$$\operatorname{dist}\big((\delta(x)+\rho)/(n+r), \partial\mathcal{M}\big) \geq K n^{-1/(2\eta)}$$

where δ is a generalized distance from the origin and $\mathcal{M}$ is the convex envelop of the set

$$\big\{\delta(x) \in P^+ : p(O,x) > 0\big\}.$$

In particular,

Theorem. *For any $\epsilon > 0$ small enough*

$$p_n(x) \asymp n^{-r/2-|\Phi_0^+|} \rho^n P_{\delta(x)}(0) \exp\big(-n\phi\big(\delta(x)/n\big)\big)$$

uniformly on $\{x \in \operatorname{supp}\, p_n : \operatorname{dist}(\delta(x)/n, \partial\mathcal{M}) \geq \epsilon\}$.

The basic tool in the study of isotropic random walks is the spherical Fourier transform. In the 1970s, Macdonald developed spherical harmonic analysis for groups of p-adic type. An application of the spherical Fourier transform results in an oscillatory integral which we analyse by the steepest descent method. Thanks to some geometric properties of the support of the spherical Fourier transform of p, the integral can be localized. Therefore, the proof reduces to establishing the asymptotic behaviour, as n approaches infinity, of

$$I_n(x) = \int\limits_{|u|\leq\epsilon} e^{n\varphi(x,u)} f(x,u) du,$$

uniformly with respect to $x \in \mathfrak{a}_+$ where $\mathfrak{a}_+$ is the Weyl chamber of the underlying root system. If x lies on the wall of $\mathfrak{a}_+$ then the function $\varphi(x,\cdot)$ retain symmetries in the directions orthogonal to the wall. Close to the wall we take advantage of this by expanding I_n into its Taylor series and using combinatorial methods to identify remaining cancellations.