Multiplicity free induction and orthogonal polynomials

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Given a compact Gelfand pair (G, K) of rank one, the induction of the trivial K-representation to G decomposes multiplicity free as a G-module. In the harmonic analysis on G/K this is reflected in the fact that the algebra of G-invariant differential operators on G/K is commutative. This, in turn, brings a family of Jacobi polynomials into the game, as simultaneous eigenfunctions for the G-invariant differential operators on G/K.

There are more irreducible K-representations π^K whose induction to G decomposes multiplicity free as a G-module. In fact, the triples (G, K, π^K) with this property have been classified recently. In the case that (G, K) is of rank one, such a triple (G, K, π^K) gives rise to a family of matrix valued orthogonal polynomials with properties that are similar to those of a family of Jacobi polynomials. For some higher rank examples we find similar families of orthogonal polynomials, now in several variables.

In this talk I will report on this research and on possible applications to the harmonic analysis on homogeneous vector bundles over non-compact homogeneous spaces, that are subject to similar multiplicity free regime.