Connecting distributive homology and Khovanov homology, via Yang-Baxter operators

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Session: Knot Theory

One of the most important development of the last 15 years was Khovanov's categorification of the Jones polynomial of links. In parallel, the theory of homology of racks and quandles and its applications to knot theory was developed. We propose and develop here an idea how to connect distributive (e.g. rack or quandle) homology and Khovanov homology, via Yang-Baxter operators (the fundamental tool of statistical physics). As a special case of my ideas (technical but easy to visualize), I will show how to construct a q-polynomial for rooted trees (or more generally finite plane poset). This becomes a tool to analyze the Kauffman bracket skein modules of 3-manifolds (starting from a cable crossing). For a plane tree T with a root v, we define a polynomial $Q(T)$ by a recursive relation: $Q(T)$ is equal to the sum over all leaves, v_i, of T of the polynomials $Q(T - v_i)$ with the coefficient $q^{r(v_i)}$, where $r(v_i)$ is the number of edges of T to the right of v_i. We normalize $Q(T)$ to be 1 at the root. We prove several properties of $Q(T)$, e.g. that it is tree invariant, that is, does not depend on a plane embedding. We relate $Q(T)$ to Gauss polynomials (q-binomial coefficients). The application of $Q(T)$ to a lattice crossing will be presented in a join paper with M. Dąbkowski and C. Li.