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Of copulas, quantiles, ranks, and spectra: an L_1-approach to spectral analysis

Holger Dette

Ruhr-Universität Bochum, Germany
holger.dette@ruhr-uni-bochum.de

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In this paper, we present an alternative method for the spectral analysis of a univariate, strictly stationary time series $\{Y_t\}_{t\in\mathbb{Z}}$. We define a "new" spectrum as the Fourier transform of the differences between copulas of the pairs (Y_t, Y_{t-k}) and the independence copula. This object is called a *copula spectral density kernel* and allows to separate the marginal and serial aspects of a time series. We show that this spectrum is closely related to the concept of quantile regression. Like quantile regression, which provides much more information about conditional distributions than classical location-scale regression models, copula spectral density kernels are more informative than traditional spectral densities obtained from classical autocovariances. In particular, copula spectral density kernels, in their population versions, provide (asymptotically provide, in their sample versions) a complete description of the copulas of all pairs (Y_t, Y_{t-k}). Moreover, they inherit the robustness properties of classical quantile regression, and do not require any distributional assumptions such as the existence of finite moments. In order to estimate the copula spectral density kernel, we introduce rank-based Laplace periodograms which are calculated as bilinear forms of weighted L_1-projections of the ranks of the observed time series onto a harmonic regression model. We establish the asymptotic distribution of those periodograms, and the consistency of adequately smoothed versions. The finite-sample properties of the new methodology, and its potential for applications are briefly investigated by simulations and a short empirical example. We also discuss the advantages of a localized version of the method.