

## Copulas, hairpins and tail dependence

**Fabrizio Durante**

Free University of Bozen-Bolzano, Italy  
[fabrizio.durante@unibz.it](mailto:fabrizio.durante@unibz.it)

*The talk is based on a joint work with Juan Fernández-Sánchez and Wolfgang Trutschnig*

*Session: 22. Multivariate stochastic modelling in finance, insurance and risk management*

In this talk, we focus on the class of  $d$ -dimensional copulas ( $d \geq 2$ ) whose support is contained in a hairpin set, which is, in dimension two, the union of the graphs of two increasing homeomorphisms  $h$  and  $g$ . Interestingly, however, the notion of two-dimensional hairpin allows for two different extensions to the general multivariate setting, which we call sub-hairpin and super-hairpin, respectively. We study existence and uniqueness of  $d$ -dimensional copulas whose support is contained in a sub- (or super-) hairpin and extend various results about bivariate copulas to the general multivariate setting. In particular, we show that each copula with hairpin support is necessarily an extreme point of the convex set of all  $d$ -dimensional copulas and introduce a simple transformation that maps copulas with sub-hairpin support to copulas with super-hairpin support and vice-versa.

Two are the main applications of the presented results. First, sub- (or super-) hairpin copulas are strictly related to some constructions of copulas with fixed diagonal section. As such, they are related to tail dependence and tail dependence coefficients. Second, since copulas with hairpin support have a singular component, they can be used in credit risk models in order to describe the possible occurrence of joint defaults.