Characterizing exchangeable fatal shock models and a link to self-decomposability on the real line

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Fatal shock models have a long tradition in reliability theory and have frequently been applied in finance and insurance. A milestone publication is [Marshall, Olkin (1967)], where a stochastic model with independent and exponential fatal shocks is constructed and analytically characterized. Since then, this approach has been generalized in many directions such as alternative shock distributions, dependent shocks, etc. The technical difficulties to circumvent are – in the d-variate case – the exponentially increasing number of shocks, exposing one to difficult combinatorial problems. In the exchangeable case our results show that the resulting dependence structures are given by functions $C : [0,1]^d \to [0,1], d \geq 2$, of the form

$$C(u_1, \ldots, u_d) = \prod_{k=1}^{d} g_k(u_{(k)}), \tag{1}$$

where $g_1 = \mathrm{id}_{[0,1]}$, the functions $g_k : [0,1] \to [0,1]$ satisfy $g_k(1) = 1, k = 2, \ldots, d$, and $u_{(1)} \leq u_{(2)} \leq \ldots \leq u_{(d)}$ is the ordered list of $u_1, \ldots, u_d$. Necessary and sufficient conditions on g_k, $k = 2, \ldots, d$, such that C defines a copula are derived, which is the main result of [Mai, Schenk, Scherer (2014a)]. Letting d tend to infinity provides a link to conditionally iid models. This provides a probabilistic tool to relate analytical properties of C to additive processes, serving as mixture variable, and especially Sato processes as an important sub-family. This link, finally, provides two new characterizations of self-decomposability on the real line, as shown in [Mai, Schenk, Scherer (2014b)].

References

[Marshall, Olkin (1967)] A.W. Marshall, I. Olkin, A multivariate exponential distribution, *Journal of the American Statistical Association* **62** (1967) pp. 30–44.

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