Shadow prices for markets with proportional transaction costs

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We study shadow price in general discrete time finite horizon setting. Usual shadow price (we call it strong shadow price) depends on initial portfolio position and is the price under which portfolio strategies and value of the functional (utility functions of consumption) are the same as in the market with bid and ask prices. By its definition it does not depend on the current values of the portfolio position but of course is different for other initial portfolio positions. In the paper we introduce a notion of weak shadow price, which his constructed at each time for all possible portfolio values but depends on the current value of the portfolio position, and for which the optimal value and optimal strategies are the same as in the market with bid and ask prices. From the point of view of dynamic programming our construction is more natural and allows us to perform it stepwise. The main assumptions we impose are: conditional full support, absence of short selling and strict concavity of utility functions. Studying general Bellman equations we obtain a number of convenient properties of buying, selling and non transaction regions. As a result in general setting we construct weak shadow price, which is defined first locally and then globally. Under differentiability of the value function (which is in particular in the case of logarithmic or power utility functions) weak shadow price is defined in a unique way. In general case it may be non unique within non transaction region. With the use of weak shadow price we construct strong shadow price. We show that construction strong shadow price is naturally associated with the notion of weak shadow price. Extensions of the construction for infinite horizon discounted utility functional are also considered. The presentation is based on joint works with my PhD student T. Rogala.