Balance laws with fluxes discontinuous in the unknown and the spatial variable

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We are interested in a scalar balance law in an arbitrary dimension d with a discontinuous/multivalued flux and dissipative source term

$$u_t + \operatorname{div} \Phi(x,u) \ni f(t,x,u) \quad \text{on } \mathbb{R}_+ \times \mathbb{R}^N, \tag{1}$$

$$u(0,\cdot) = u_0 \quad \text{on } \mathbb{R}^N. \tag{2}$$

The presented framework includes the fluxes which are discontinuous in the spatial variable x and in the unknown function u. Under some additional hypothesis on the structure of possible discontinuities, we formulate an appropriate notion of entropy solution and establish its existence and uniqueness. The structure of the flux function corresponds to the one proposed by [4] for the case of fluxes discontinuous in x.

We partially follow the idea of entropy measure valued solutions tools and the method of doubling the variables, but on the level of measure valued solutions, see [1, 2, 3, 4]. The starting point in this framework is the definition of entropy measure valued solutions and the so-called contraction principle, which is satisfied by entropy measure valued solutions. The essential part of the proof is showing the comparison principle for entropy weak solutions and using the semi-Kružkov entropies, cf. [4].

References

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