Regularity results for solutions to heat equation with the initial condition in Orlicz–Slobodetskii space

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We study the initial problem for heat equation:

$$\begin{cases} u_t(x,t) = \Delta_x u(x,t) & \text{in } \Omega \times (0,T), \\ u(x,0) = u_0 & \text{for } x \in \Omega, \end{cases} \tag{1}$$

where $\Omega \subseteq \mathbb{R}^n$ is a Lipschitz boundary domain, u lies in the completion of $C_0^\infty(\Omega)$ in certain Orlicz-Slobodetski type space $Y^{R_1,R_2}(\Omega)$ which is defined in the following way. Let R_1, R_2 be the possibly different Orlicz spaces. By $Y^{R_1,R_2}(\Omega)$ and denote the space consisting of all $u \in L^{R_1}(\Omega)$, for which the seminorm

$$I^{R_2}(u,\Omega) := \int_\Omega \int_\Omega R_2\left(\frac{|u(x)-u(y)|}{|x-y|}\right) \frac{dxdy}{|x-y|^{n-1}} \tag{2}$$

is finite.

We prove that if R satisfies certain assumptions and $u_0 \in Y^{R,R}(\Omega)$, then the solution u of our heat equation lies in the Orlicz-Sobolev space $W^{1,R}(\Omega \times (0,T))$, which by definition fulfills the requirement: u, together with its all first order partial derivatives belongs to Orlicz space $L^R(\Omega \times (0,T))$. The typical representant of the admissible Orlicz space is $L(LogL)^\alpha$ where $\alpha \geq 1$.

More generally, when R_1, R_2 are possibly different but satisfy certain compatibility condition due to Kita, we obtain regularity results involving the initial condition $u_0 \in Y^{R_1,R_2}(\Omega)$ and $u \in W^{1,R_1}(\Omega \times (0,T))$, where R_1 can essentially dominate R_2.

Lecture will be based on the following issues:

References

[1] A. Kalamajska, M. Krbec, *On solutions to heat equation with the initial condition in Orlicz-Slobodetskii space*, to appear in Proc. Roy. Soc. Edinburgh Sec. A Mathematics.

[2] A. Kalamajska, M. Krbec, *Orlicz regularity theory for solutions to heat equation with the initial condition in Orlicz-Slobodetskii space*, in preparation.