Long-time behavior for a simplified Keller-Rubinov model for Liesegang rings in the fast reaction limit

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Liesegang rings are regular patterns in a chemical precipation reaction which typically follow power laws in the spacing of width and distance. Among several mathematical models, the Keller–Rubinov model [3] is a reaction-diffusion equation with a super-saturation threshold in the reaction term. Hilhorst *et al.* [2, 1] study the fast reaction limit in which, among other assumptions, the reaction rate of the initial step of the mechanism is assumed to be infinitely fast. As a result, the Keller–Rubinov model reduces to a single reaction-diffusion equation with a singular driving term and a reaction term involving memory and thresholding.

As in [1], we consider the one-dimensional situation, which we shall refer to as the HHMO model. Noting that the HHMO model possesses a self-similar solution that is explicitly expressable in terms of special fuctions, we present numerical evidence as well as a mechanism derived from a simplifed version of the system which indicate that the self-similar profile is "super-attracting" in the sense that solutions to the HHMO model tend to the self-similar profile in a finite time. We then re-interpret the self-similar profile as a precipitation density function.

References

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