Renormalized solutions of nonlinear parabolic problems in generalized Musielak–Orlicz spaces

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We are interested in existence and uniqueness of renormalized solutions to the nonlinear initial-boundary value problem

$$\begin{aligned} \partial_t \beta(x,u) - \operatorname{div}(a(x,Du) + F(u)) &= f \text{in } Q_T = (0,T)\times\Omega \\ u &= 0 \text{on } \Sigma_T = (0,T)\times\partial\Omega \\ \beta(\cdot, u(0,\cdot)) &= b_0 \text{on } \Omega, \end{aligned}$$

where

- $\beta\colon \Omega\times\mathbb{R}\to\mathbb{R}$ is a monotone single-valued Carathéodory function,
- $F\colon \mathbb{R}\to\mathbb{R}^N$ is a locally Lipschitz continuous function,
- b_0 and f are integrable given data, and
- the Carathéodory vector field $a\colon \Omega\times\mathbb{R}^N\to\mathbb{R}^N$ is monotone w.r.t. $\xi\in\mathbb{R}^N$ and satisfies generalized growth and coerciveness conditions of the form $a(x,\xi)\cdot\xi \geq c_a(M(x,\xi) + M^*(x,a(x,\xi)) - a_0(x) \quad \text{a.e. } x\in\Omega, \forall\xi\in\mathbb{R}^N,$

with $c_a > 0$, $a_0 \in L^1(\Omega)$, $M : \Omega\times\mathbb{R}^N\to\mathbb{R}$ being a generalized $\mathcal{N}$-function with complementary function M^*. Our setting includes parabolic problems involving the $p(x)$- and also the anisotropic $p = (p_1,\dots,p_N)$-Laplacian with variable exponents essentially larger than 1.

The appropriate functional setting involves generalized Musielak-Orlicz spaces $L_M(\Omega;\mathbb{R}^N)$ which, in general, are neither separable nor reflexive. Therefore, classical monotonicity and truncation techniques have to be appropriately adapted to the non-reflexive and non-separable functional setting.