A swelling model: existence, basic properties

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The talk is based on a joint work with Marta Lewicka (University of Pittsburgh)

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We study the evolutionary model in the description of morphogenesis of an elastic body Ω exhibiting residual strain at free equilibria. The model consists of the balance of linear momentum and the diffusion law:

$$u_{tt} - \operatorname{div}\left(\frac{1}{f(\phi)} DW(\frac{1}{f(\phi)}\nabla u)\right) = 0$$

$$\phi_t = \Delta(-\frac{f'(\phi)}{f(\phi)^2}\langle DW(\frac{1}{f(\phi)}\nabla u) : \nabla u\rangle + \frac{\partial F}{\partial \phi})$$

written in terms of the deformation u of Ω ($u : \Omega \to \mathbb{R}^N$), and the growth (swelling) agent density ϕ. The basic structure assumptions are:

$$W(X) \geq c|X - Id|^2, \quad |DW(X)| \leq c|X - Id|, \quad D^2W(X)M \otimes M \geq c|Sym\, M|^2$$

for X close to Id. Functions f and F are sufficiently smooth and obey natural conditions. The basic information is given by the energy inequality

$$\frac{d}{dt}\int \left(\frac{1}{2}u_t^2 + W(\frac{1}{f(\phi)}\nabla u) + F(\phi)\right) dx \leq 0.$$

We investigate the issue of well posedness of the system. The key difficulty is located in the hyperbolic character of the system and very high nonlinearities of the second equation. The straightforward approaches do not lead to the a priori estimate. To find the required information about the solutions we have to analyze the structure of the nonlinearities.

We plan to point several interesting properties and possible generalizations of the studied system, too.

The talk will base on results of the joint work with Marta Lewicka [1].

References

[1] M. Lewicka, P.B. Mucha, *The evolutionary swelling model*, in preparation.