A Canonical Extension of Korn's First Inequality to H(Curl) motivated by Gradient Plasticity with Plastic Spin

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We prove a Korn-type inequality in $\dot{\mathrm{H}}(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$ for tensor fields P mapping Ω to $\mathbb{R}^3$. More precisely, let $\Omega \subset \mathbb{R}^3$ be a bounded domain with connected Lipschitz boundary $\partial\Omega$. Then, there exists a constant $c > 0$ such that

$$c\|P\|_{L^2(\Omega,\mathbb{R}^{3\times 3}} \leq \|\operatorname{sym} P\|_{L^2(\Omega,\mathbb{R}^{3\times 3}} + \|\operatorname{Curl} P\|_{L^2(\Omega,\mathbb{R}^{3\times 3}} \tag{1}$$

holds for all tensor fields $P \in \dot{\mathrm{H}}(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$, i.e., all $P \in \dot{\mathrm{H}}(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$ with vanishing tangential trace on $\partial\Omega$. Here, rotation and tangential trace are defined row-wise. For compatible P, i.e., $P = \nabla v$ and thus $\operatorname{Curl} P = 0$, where $v \in \mathrm{H}^1(\Omega,\mathbb{R}^3)$ are vector fields having components v_n, for which ∇v_n are normal at $\partial\Omega$, the presented estimate (1) reduces to a non-standard variant of Korn's first inequality, i.e.,

$$c\|\nabla v\|_{L^2(\Omega,\mathbb{R}^{3\times 3}} \leq \|\operatorname{sym} \nabla v\|_{L^2(\Omega,\mathbb{R}^{3\times 3}}.$$

On the other hand, for skew-symmetric P, i.e., sym $P = 0$, (1) reduces to a non-standard version of Poincaré's estimate. Therefore, since (1) admits the classical boundary conditions our result is a common generalization of the two classical estimates, namely Poincaré's resp. Korn's first inequality. Applications to infinitesimal gradient plasticity with plastic spin are given.