Approximation of evolution semigroups generated by pseudo-differential operators via Feynman formulae

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Session: 25. Nonlocal Phenomena: Levy processes and related operators

Evolution semigroups $e^{-t\widehat{H}}$ generated by pseudo-differential operators $\widehat{H}$ are considered. The operators $\widehat{H}$ are obtained from a given function $(q,p) \mapsto H(q,p)$ (which is called a symbol of $\widehat{H}$) by some linear procedure (which is called a quantization). We consider a class of such procedures, parameterized by a number $\tau \in [0,1]$; this class includes $qp-$, $pq-$ and Weyl quantizations. We consider $q, p \in \mathbb{R}^d$, continuous negative definite with respect to the "momentum"-variable p functions H and semigroups $e^{-t\widehat{H}}$ on Banach spaces $C_\infty(\mathbb{R}^d)$ and $L^1(\mathbb{R}^d)$. Our approach is to approximate the semigroup $e^{-t\widehat{H}}$ (for a given procedure of quantization) by a family of pseudo-differential operators $\widehat{e^{-tH}}$ obtained by some (the same or different) procedure of quantization from the function e^{-tH}. By this approach the semigroups $e^{-t\widehat{H}}$ are represented as limits of n-fold iterated integral operators with elementary kernels when n tends to infinity. Such representations are called Feynman formulae. They can be used for direct computations and simulation of corresponding stochastic processes. On the other hand, the limits of iterated integrals in the obtained Feynman formulae coincide with some Feynman–Kac formulae, i.e.with functional integrals with respect to probability measures generated by underlying stochastic processes. Moreover, these limits can be interpreted as Feynman path integrals.

References

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