Decay of eigenfunctions for nonlocal Schrödinger operators

Kamil Kaleta

Wroclaw University of Technology and University of Warsaw, Poland
kamil.kaleta@pwr.edu.pl; kkaleta@mimuw.edu.pl

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The spatial decay of eigenfunctions at infinity for Schrödinger operators $H = -\Delta + V$, where V is a suitably chosen potential, has been widely studied for many years. A basic interest in this property is that it tells of how well a quantum particle described by H is localized in physical space. When V is a pinning potential, i.e., $V(x) \to \infty$ as $|x| \to \infty$, then the decay is known to be typically faster than exponential. Questions about the decay of eigenfunctions, motivated by the problems in a relativistic quantum mechanics, also appear in the case of the so-called nonlocal Schrödinger operators

$$H = -L + V,$$

where L is a nonlocal operator which is the generator of the jump Lévy process.

I will present the recent results on the pointwise bounds at infinity of the eigenfunctions of H for a wide class of operators L and signed potentials V. These estimates explicitly depend on the density of the Lévy measure of the process generated by L and the growth of V at infinity. For the ground state eigenfunction (which is known to be strictly positive) they are even two-sided and sharp. Our methods are mainly probabilistic (stochastic Feynman-Kac-type representation of the semigroup e^{-tH}) and are based on a precise analysis of jumps of the process and some specific self-improving estimates iterated infinitely many times. I will also discuss some interesting consequences and applications of these results: the properties of domination (of the semigroup e^{-tH} and other eigenfunctions) by the ground state eigenfunction, the asymptotic behaviour of the semigroup e^{-tH} for large t (intrinsic ultracontractivity-type properties), the asymptotic behaviour of paths of the related ground state-transformed jump processes (integral tests of the Kolmogorov-type, LILs, etc.).

References

[1] K. Kaleta, J. Lőrinczi, *Pointwise eigenfunction estimates and intrinsic ultracontractivity-type properties of Feynman-Kac semigroups for a class of Lévy processes*, Ann. Probab., to appear, 2014.