Weak convergence of finite element approximations of linear stochastic evolution equations with additive Lévy noise

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We consider linear stochastic evolution equations of the form

$$\mathrm{d}X(t) + AX(t)\,\mathrm{d}t = B\,\mathrm{d}Z(t), \quad t \in [0,T], \quad X(0) = X_0,$$

where the solution process X takes values in a Hilbert space H, $-A$ is the generator of a strongly continuous operator semigroup on H, B is an H-valued linear and bounded operator, and Z is an infinite dimensional Lévy process. For instance, the stochastic heat equation and the stochastic wave equation with additive Lévy noise can be written in this abstract form. Given an approximation $\tilde{X}(T)$ of $X(T)$, we are interested in estimating the weak error $\big|\mathbb{E}\big(G(\tilde{X}(T)) - G(X(T))\big)\big|$ for suitable test functions $G : H \to \mathbb{R}$.

We use Itô's formula and the backward Kolmogorov equations associated to certain auxiliary processes to derive a general error representation formula. The formula is then applied to spatially semidiscrete finite element methods for the stochastic heat equation and the stochastic wave equation.

Explicit rates of convergence are obtained, depending on the type of the considered equation, the assumptions on Z and G, and the order of the finite element method. The rates of weak and strong convergence are compared.