Exotic smoothness, foliations and torsion on 4-manifolds

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The geometry of open exotic 4-manifolds like exotic \mathbb{R}^4 or $S^3 \times \mathbb{R}$ is quite mysterious but important from the point of physics. In this talk, the relation of exotic smoothness structures to foliations is discussed. The central invariant in this context is the Godbillon-Vey invariant which will be related to the parameter of the DeMichaelis-Freedman radial family of small exotic \mathbb{R}^4 . The leaf space of these foliations is a factor *III* von Neumann algebra in the sense of Connes. The origin of this algebra can be understood by analyzing wildly embedded submanifolds like the topologically embedded 3-sphere in the exotic $S^3 \times \mathbb{R}$. From the geometrical point of view, these foliations are related to hyperbolic geometry (the non-compact counterpart of non-positive scalar curvature results for compact exotic 4-manifolds). Finally, the appearance of torsion in exotic 4-manifolds is also discussed.