Classification of totally umbilical foliations in hyperbolic spaces

Maciej Czarnecki

Uniwersytet Łódzki, Poland maczar@math.uni.lodz.pl

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On hyperbolic manifolds neither totally geodesic (many authors) nor totally umbilical (Langevin–Walczak) foliations of codimension 1 exist. Totally geodesic foliations of the real hyperbolic space \mathbb{H}^n were classified by Ferus using curvature of orthogonal transversal. The next geometric step towards it was done by Browne who weakened these conditions. Lastly, Lee and Yi gave a boundary classification concerning centers of leaf ideal boundaries.

Using the Sitter space Λ^{n+1} in the Lorentz space understood as the space of (n-1)-spheres in S^n we give a conformal classification of totally umbilical codimension 1 foliations of \mathbb{H}^n (in particular, for totally geodesic).

We prove that a curve in Λ^{n+1} represents a foliations of (a domain in) \mathbb{H}^n iff its tangent vector belongs everywhere to a boosted time cone (Shadok cone) and give geometric interpretation in terms of the mean curvature of leaves and geodesic curvature of an orthogonal transversal.

References

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