

Harmonious Coloring of Hypergraphs

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The talk is based on the joint work with Sebastian Czerwiński,¹ Jarosław Grytczuk,² and Paweł Rzżewski.³

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A *harmonious* coloring of a hypergraph is a coloring of its vertices such that

- two different vertices contained in common hyperedge have different colors,
- sets of colors of all vertices from two different hyperedges are different sets.

We prove that every k -uniform hypergraph, with maximal degree Δ and with m hyperedges, is harmoniously colored by

$$O\left(\frac{k}{k-1} \sqrt[k]{(k-1)k!\Delta m} + \Delta^2 + (k-1)\Delta\right)$$

colors. This is almost tight, since the obvious lower bound is of order $\Omega(\sqrt[k]{k!m})$. The proof uses *entropy compression* argument – a novel method inspired by the algorithmic version of the Lovász Local Lemma due to Moser and Tardos. We will also discuss several related problems, for instance *legitimate* coloring of hypergraphs, *strong* coloring of graphs, etc.

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