Irregularity in graphs

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How to define an irregular graph? This very basic question was posed and exploited in 1988 as a title of a paper by Chartrand, Erdős and Oellermann. The confusion originates from the well known fact that no antonym of a regular graph, understood as a graph whose all vertices have pairwise distinct degrees, exists, except for the trivial 1-vertex case. This limitation does not concern multigraphs though. Consequently, the following extension of these research was developed as an attempt of designing a graph invariant measuring the level of 'irregularity' of a graph. Suppose that given a simple graph $G = (V, E)$ we are allowed to multiply some of its edges. How small can be the largest necessary multiplicity of an edge so that we are able to construct an *irregular multigraph* of G, i.e., a multigraph with pairwise distinct vertex degrees? This value was named the *irregularity strength* of G, denoted by $s(G)$. Alternatively, one may consider a (colouring) function $c : E \to \{1, 2, \ldots, k\}$, assigning every edge an integer corresponding to its multiplicity in a desired multigraph. The least k so that such colouring exists attributing every vertex of G a distinct sum of incident colours is then equal to $s(G)$.

This issue was a cornerstone of many other combinatorial questions and colouring problems including e.g. 1–2–3 Conjecture and Zhang's Conjecture, as well as some problems of a more structural flavour, like graph decompositions into locally irregular subgraphs, or complexity problems concerning these.

As appeared this field also constitutes a natural environment for nice applications of the probabilistic method, and provides some observations on random graphs themselves. A few of its consequential results reach far beyond this particular branch of graph theory.

A number of key questions of the field shall be presented during the talk, accompanied by representative results concerning these.