

Cycles in triangle-free graphs of large chromatic number

Benny Sudakov

ETH Zürich, Switzerland

benjamin.sudakov@math.ethz.ch

The talk is based on the joint work with Alexandr Kostochka and Jacques Verstraëte.

Session: 27. Probabilistic and Extremal Combinatorics

More than twenty years ago Erdős conjectured that a triangle-free graph G of chromatic number $k \geq k_0(\varepsilon)$ contains cycles of at least $k^{2-\varepsilon}$ different lengths as $k \rightarrow \infty$. In this paper, we prove the stronger fact that every triangle-free graph G of chromatic number $k \geq k_0(\varepsilon)$ contains cycles of $(\frac{1}{64} - \varepsilon)k^2 \log k$ consecutive lengths, and a cycle of length at least $(\frac{1}{4} - \varepsilon)k^2 \log k$. As there exist triangle-free graphs of chromatic number k with at most roughly $4k^2 \log k$ vertices for large k , these results are tight up to a constant factor. We also give new lower bounds on the circumference and the number of different cycle lengths for k -chromatic graphs in other monotone classes, in particular, for clique-free graphs and graphs without odd cycles.