Hamilton saturated hypergraphs of essentially minimum size

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For $1 \leq \ell < k$, an ℓ-*overlapping cycle* is a k-uniform hypergraph in which, for some cyclic vertex ordering, every edge consists of k consecutive vertices and every two consecutive edges share exactly ℓ vertices.

A k-uniform hypergraph H is ℓ-*Hamiltonian saturated* if H does not contain an ℓ-overlapping Hamiltonian cycle but every hypergraph obtained from H by adding one more edge does contain such a cycle. Let $\mathrm{sat}(n,k,\ell)$ be the smallest number of edges in an ℓ-Hamiltonian saturated k-uniform hypergraph on n vertices. In the case of graphs Clark and Entringer [1] proved in 1983 that $\mathrm{sat}(n,2,1) = \lceil \frac{3n}{2} \rceil$.

For hypergraphs with $k \geq 3$ it seems to be quite hard to obtain such precise results. Therefore, the emphasis is put on the order of magnitude of $\mathrm{sat}(n,k,\ell)$. We proved that for $k \geq 3$ and $\ell = 1$ as well as for all $0.8k \leq \ell \leq k-1$

$$\mathrm{sat}(n,k,\ell) = \Theta(n^{\ell}), \tag{1}$$

see [2, 3]. Recently, we have got also some partial results: $\mathrm{sat}(n,k,\ell) = O(n^{(k+\ell)/2})$ and, in the smallest open case, $\mathrm{sat}(n,4,2) = O(n^{14/5})$.

References

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