Bounds on equivariant Betti numbers for symmetric semi algebraic sets

Cordian Riener

Aalto University, Finland
cordian.riener@aalto.fi

The talk is based on the joint work with Saugata Basu

Session: Real algebraic geometry, applications and related topics

Let R be a real closed field. We prove upper bounds on the equivariant Betti numbers of symmetric algebraic and semi-algebraic subsets of R^k. More precisely, we prove that if $S \subset R^k$ is a semi-algebraic subset defined by a finite set of s symmetric polynomials of degree at most d, then the sum of the $S_k$equivariant Betti numbers of S with coefficients in $\mathbf{Q}$ is bounded by $s^5 d(kd)^{O(d)}$. Unlike the well known classical bounds due to Oleinik and Petrovskii, Thom and Milnor on the Betti numbers of (possibly non-symmetric) real algebraic varieties and semi-algebraic sets, the above bound is polynomial in k when the degrees of the defining polynomials are bounded by a constant. Moreover, our bounds are asymptotically tight. As an application we improve the best known bound on the Betti numbers of the projection of a compact semi-algebraic set improving for any fixed degree the best previously known bound for this problem due to Gabrielov, Vorobjov and Zell.