On cusps and swallowtails of real polynomial mappings

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Let M be an oriented n-manifold, where $n = 2, 3$. For a generic $f \in C^\infty(M, R^n)$, there is a discrete set $S(f)$ of critical points consisting of cusp points if $n = 2$, or swallowtail points if $n = 3$.

In that case, at any $p \in S(f)$ there exists a well-oriented coordinate system centered at p, and a coordinate system centered at $f(p)$, such that locally f has the form

$$f_\pm(x, y) = (\pm x, xy + y^3) \quad \text{if } n = 2, \tag{1}$$

$$f_\pm(x, y, z) = (\pm xy + x^2z + x^4, y, z) \quad \text{if } n = 3, \tag{2}$$

so one may associate with p a sign $I(f, p) \in \{\pm 1\}$. In the planar case the sign of a cusp equals the local topological degree of $f : (M, p) \to (R^2, f(p))$. A geometric definition of the sign associated with a swallowtail was recently introduced by Goryunov [1].

We shall show how to compute the number of points in $S(f)$ having the positive/negative sign in the case where $f\colon R^n \to R^n$ is a polynomial mapping in terms of signatures of quadratic forms.

References

[1] V. Goryunov, *Local invariants of maps between 3-manifolds*, Journal of Topology 6 (2013), 757–776.