Semidefinite representation of convex sets

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A spectrahedron in $\mathbb{R}^n$ is the solution set of a linear matrix inequality $A_0 + \sum_{i=1}^n x_i A_i \succeq 0$. Linear projections of spectrahedra are called semidefinitely representable sets, or sdp sets. These sets are the feasible sets for semidefinite optimization, which is known to perform very efficiently in polynomial time. Every semidefinitely representable set is semialgebraic and convex, but no other restrictions are known. A basic technique for constructing sdp approximations to a given convex set, called the relaxation method, is due to Lasserre. It relates the problem to weighted sums of squares representations of nonnegative polynomials. Nemirovsky (2006) asked for a characterization of semidefinitely representable sets. Helton and Nie (2010) conjectured that every convex semialgebraic set should be semidefinitely representable, and provided much evidence for this conjecture. After an introduction to the problem and an overview of known results, I shall try to explain the main steps of the proof of the conjecture in dimension two.