Anosov diffeomorphisms, infra-nilmanifolds and the abelianized holonomy representation

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An infra-nilmanifold is a manifold which is constructed as a quotient space $\Gamma\backslash G$ of a simply connected nilpotent Lie group G, where Γ is a discrete group acting properly discontinuously and cocompactly on G via so called affine maps. The manifold $\Gamma\backslash G$ is said to be modeled on the Lie group G. The group Γ fits in a short exact sequence

$$1 \to N \to \Gamma \to F \to 1. \tag{1}$$

where N is a lattice in G and F is a finite group. Being a lattice in G, we know that the group N is a finitely generated torsion free nilpotent group.

For any nilpotent group N, we can consider the group $\sqrt{[N,N]}$, which is the inverse image under the natural projection $p : N \to N/[N,N]$ of the torsion subgroup of $N/[N,N]$. So when N is finitely generated, $[N,N]$ is of finite index in $\sqrt{[N,N]}$ and $N/\sqrt{[N,N]} \cong \mathbb{Z}^k$ for some k.

Returning to the study of infra-nilmanifolds, the short exact sequence (1) induces a representation

$$\varphi : F \to \mathrm{Aut}(N/\sqrt{[N,N]}) \cong \mathrm{GL}_k(\mathbb{Z}),$$

which we call the abelianized holonomy representation of Γ (or of the corresponding infra-nilmanifold). We can also view this representation as a representation $F \to \mathrm{GL}_k(\mathbb{Q})$ and then we talk about the rational abelianized holonomy representation.

The class of infra-nilmanifolds is conjectured to be the only class of closed manifolds allowing an Anosov diffeomorphism. However, it is far from obvious which of these infra–nilmanifolds actually do admit an Anosov diffeomorphism.

In this talk we will explain that for an infra-nilmanifold which is modeled on a free nilpotent Lie group the existence problem can be completely solved and depends only on the rational holonomy representation of the infra-nilmanifold.