Smooth Group Actions on Homotopy Complex Projective Spaces

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The fixed point sets of smooth finite group actions on Euclidean spaces, disks and spheres have been studied extensively. In particular, a complete description of the fixed point sets of such actions on disks has been obtained in [5] and [4]. In the case of actions on spheres, a similar characterisation has been found for finite perfect groups (see [2] and [3]).

In this talk, we present a description of the fixed point sets of smooth actions of finite perfect groups on manifolds homotopy equivalent to complex projective spaces. In [1], we show that the following theorem holds.

Theorem *Let G be a finite perfect group with an element not of prime power order. Let F be a closed smooth manifold with an even dimensional connected component. If F is diffeomorphic to the fixed point set of a smooth action of G on a sphere, then F can be realised as the fixed point set of a smooth action of G on a complex projective space.*

In particular, if G has a $2n$-dihedral sub-quotient for a composite natural number n, then any closed smooth manifold is diffeomorphic to the fixed point set of a smooth action of G on a complex projective space.

If $G = A_5$ and F is a closed smooth manifold such that $[\tau_F] \in \operatorname{Tor} \widetilde{KO}(F)$ and all connected components of F are of the same even dimension, we show that F can be realised as the fixed point set of a smooth action of G on a closed smooth manifold *homotopy equivalent* to a complex projective space.

References

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