Parameters for defining characteristic representations and counting semisimple classes

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The talk is based on the joint work with Olivier Brunat (Paris).

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In this talk we consider connected reductive algebraic groups G over an algebraic closure $\bar{\mathbb{F}}_p$ of a finite prime field with p elements. We assume that G is defined over a finite subfield $\mathbb{F}_q$ via a Frobenius morphism $F : G \to G$, and we are interested in the corresponding finite group of Lie type $G(q) := G^F$, the fixed points of F in G.

In the special case that G is of simply-connected type, Steinberg showed that the irreducible representations of $G(q)$ over $\bar{\mathbb{F}}_p$ are obtained as restrictions of q^l (where l is the rank of G) highest weight representations of the algebraic group G. As a consequence we get that $G(q)$ has q^l semisimple conjugacy classes.

We consider general groups G and Frobenius morphisms F given in terms of a root datum with Frobenius action on it. Starting from these data we describe a parameterization of the irreducible representations of $G(q)$ over $\bar{\mathbb{F}}_p$ (in general they are not all restrictions from the algebraic group). As an application we compute from this parameterization for all simple algebraic groups G as above and all Frobenius morphisms on G the number of semisimple classes of the corresponding finite group $G(q)$.

References

[1] O. Brunat, F. Lübeck, *On defining characteristic representations of finite reductive groups*, Journal of Algebra, Volume 395 (2013), 121-141.