Note on tangential representations on spheres

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Our targets are Smith sets for finite Oliver groups. For any finite group G, the *Smith set* $\mathrm{Sm}(G)$ consists of the differences $[U]-[V] \in \mathrm{RO}(G)$ such that for a smooth homotopy G-sphere with just two fixed points, U and V are the tangential representation spaces at the two fixed points.

In general, $\mathrm{Sm}(G)$ is not an additive subgroup of the real representation ring $\mathrm{RO}(G)$. For $[U]-[V] \in \mathrm{Sm}(G)$, U and V are isomorphic as $G_{\{p\}}$-modules for any Sylow p-subgroup $G_{\{p\}}$, for an odd prime p. Morimoto has shown that the maximal additive subgroup of $\mathrm{Sm}(G)$ is a subset of $\mathrm{RO}(G)^{\mathcal{N}_2(G)}_{\mathcal{P}(G)}$.

Here $\mathcal{P}(G)$ is the set of all subgroups of G of prime power order, $\mathcal{N}_2(G)$ is the set of all normal subgroups of G with index 1 or 2, and

$$\mathrm{RO}(G)^{\mathcal{N}_2(G)}_{\mathcal{P}(G)} = \bigcap_{P\in\mathcal{P}(G)} \ker(\mathrm{Res}^G_P \colon \mathrm{RO}(G) \to \mathrm{RO}(P)) \cap \bigcap_{N\in\mathcal{N}_2(G)} \ker(\mathrm{Fix}^N \colon \mathrm{RO}(G) \to \mathrm{RO}(G/N)).$$

So far, finite solvable Oliver groups possessing non-trivial Smith sets are not determined completely [3]. However, we know the full answer to the question for finite non-solvable groups [4]. In this talk, we give many examples of finite non-solvable groups whose Smith sets are non-trivial additive groups.

References

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