A new lower bound for the length of the hierarchy of norms

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A *norm* is a surjective function from the Baire space $\mathbb{R}$ onto an ordinal. Given two norms φ, ψ we write $\varphi \leq_{\mathrm{N}} \psi$ if φ continuously reduces to ψ. Then $\leq_{\mathrm{N}}$ is a preordering and so passing to the set of corresponding equivalence classes yields a partial order, the *hierarchy of norms*.

Assuming the axiom of determinacy (AD) the hierarchy of norms is a wellorder. The length Σ of the hierarchy of norms was investigated by Löwe in [1]; he determined that $\Sigma \geq \Theta^2$ (where $\Theta := \sup\{\alpha \mid \text{There is a surjection from } \mathbb{R} \text{ onto } \alpha\}$). In his talk "Multiplication in the hierarchy of norms", given at the ASL 2011 North American Meeting in Berkeley, Löwe presented a binary operation $\boxtimes$ on the hierarchy of norms such that for wellchosen norms φ, ψ the ordinal rank of $\varphi \boxtimes \psi$ in the hierarchy of norms is at least as big as the product of the ordinal ranks of φ and ψ, which implies that Σ is closed under ordinal multiplication and so $\Sigma \geq \Theta^\omega$.

In this talk I will note that in fact for wellchosen norms φ, ψ the ordinal rank of $\varphi \boxtimes \psi$ is exactly the product of the ranks of φ and ψ with an intermediate factor of ω_1. Furthermore using a stratification of the hierarchy of norms into initial segments closed under the $\boxtimes$-operation I will show that $\Sigma \geq \Theta^{(\Theta^\Theta)}$.

References

[1] Benedikt Löwe, *The length of the full hierarchy of norms*, Rendiconti del Seminario Matematico dell'Università e del Politecnico di Torino, vol. 63 (2005), no. 2, pp. 161–168.