

Infinite matroids and Set Theory

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Session: 32. Set Theory

Matroids provide an abstract notion of independence and have been extensively studied in the finite.

In this joint work with Nathan Bowler and Johannes Carmesin we extend the notion of a uniform matroid to the infinitary case and construct, using weak fragments of Martin's Axiom, self-dual uniform matroids on infinite sets. In 1969, Higgs showed that, assuming the Generalised Continuum Hypothesis (GCH), any two bases of a fixed matroid have the same size. We show that this cannot be proved from the usual axioms of set theory, ZFC, alone: in fact, we show that it is consistent with ZFC that there is a uniform self-dual matroid with two bases of different size.

Self-dual uniform matroids on infinite sets also provide examples of infinitely connected matroids, answering a question of Bruhn and Wollan under additional set-theoretic assumptions. While we do not know whether the existence of a self-dual uniform matroid on an infinite set can be proved in ZFC alone, we show that ZF, Zermelo-Fraenkel Set Theory without the Axiom of Choice, is not enough. Finally, we observe that there is a model of set theory in which GCH fails while any two bases of a matroid have the same size. This answers a question of Higgs.