Axiomatization of the Mathias model in terms of games

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We study the combinatorial structure of the Mathias model within the framework of descriptive set theory. We investigate a series of axioms, modeled on the CPA axiom from [1], which capture the combinatorial core of the model. They are formulated in terms of Borel sets and functions, σ-ideals on Polish spaces, games and strategies, without explicitly refering to forcing. In this way we derive a descriptive set theoretic axiomatization of the Mathias model, which makes its structure more approachable. We derive some properties of the model, in particular values of cardinal coefficients, directly from our axioms.

We also obtain a few facts about V-ultrafilters induced by reals from the generic extension via the iterated Mathias forcing. In particular, we generalize two main Propositions of [2].

References

[1] K. Ciesielski, J. Pawlikowski, *The Covering Property Axiom CPA. A combinatorial core of the iterated perfect set model.* Cambridge University Press 164, Cambridge University Press, Oxford, 2004.

[2] S. Shelah, O. Spinas *The distributivity numbers of $\mathcal{P}(\omega)/\text{fin}$ and its square*, Trans. AMS 325, 1999, 2023–2047.