Selectivity of ideals on countable sets

Piotr Zakrzewski

Institute of Mathematics, University of Warsaw, Poland
piotrzak@mimuw.edu.pl

Session: 32. Set Theory

An ultrafilter U on ω is *selective* if for every partition $\{A_n : n \in \omega\}$ of ω into sets not in U there is $S \in U$ such that $|S \cap A_n| = 1$ for every $n \in \omega$.

This well-known notion gives rise to a number of different selectivity properties of non-principal filters on ω (or equivalently: ideals on ω containing all its finite subsets). In particular, an ideal I on ω is:

- *selective* if for every partition $\{A_n : n \in \omega\}$ of ω such that no finite union of elements of the partition is in the dual filter of I there is $S \notin I$ such that $|S \cap A_n| = 1$ for every $n \in \omega$.
- *weakly selective* if for every partition $\{A_n : n \in \omega\}$ of ω such that no finite union of elements of which is in the dual filter of I and at most one element of the partition is not in I there is $S \notin I$ such that $|S \cap A_n| = 1$ for every $n \in \omega$.

Important examples of selective and weakly selective analytic ideals come from topological representations studied, among others, by Todorčević (in the case of selective ideals) and by Zapletal, Sabok and Kwela (in the case of weakly selective ideals). A striking combinatorial difference between these examples is that every weakly selective topologically representable ideal is *dense* (i.e., every infinite subset of ω has an infinite subset in the ideal) whereas by an old result of Mathias no selective analytic ideal is dense.

The talk will be devoted to such combinatorial (and topological) aspects of selective-like ideals.

References

[1] S. Todorčević, *Introduction to Ramsey Spaces*, Annals of Mathematics Studies 174, Princeton University Press, Princeton 2010.

[2] M. Sabok, J. Zapletal, *Forcing properties of ideals of closed sets*, J. Symbolic Logic 76, 2011, 1075-1095.

[3] A. Kwela, M. Sabok, *Topological representations*, Journal of Mathematical Analysis and Applications, accepted.