The logarithmic-$BMOA$ space, multipliers, and spaces of Dirichlet type

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If X and Y are two spaces of analytic functions in the unit disc $\mathbb{D}$ which are continuously contained in $\mathcal{H}ol(\mathbb{D})$, $\mathcal{M}(X,Y)$ denotes the space of multipliers from X to Y, $\mathcal{M}(X,Y) = \{g \in \mathcal{H}ol(\mathbb{D}) : fg \in Y, \quad \text{for all } f \in X\}$. The space of multipliers from X to itself will be simply denoted by $\mathcal{M}(X)$.

The spaces $\mathcal{M}(X,Y)$ have been studied for a big number of spaces X, Y. In this talk we shall concentrate our attention in the case where X and Y are spaces related with the spaces of Dirichlet type $\mathcal{D}^p_\alpha$ $(0 < p < \infty, \alpha > -1)$, $BMOA$ and the Bloch space $\mathcal{B}$.

Let us remark that the spaces $M(\mathcal{B})$ and $\mathcal{M}(BMOA)$ are known:

- $M(\mathcal{B}) = H^\infty \cap \mathcal{B}_{\log}$, where $\mathcal{B}_{\log}$ is the logarithmic Bloch space which consists of those $f \in \mathcal{H}ol(\mathbb{D})$ with $\sup_{z\in\mathbb{D}}(1-|z|^2)|f'(z)|\log\frac{1}{1-|z|^2} < \infty$.
- $\mathcal{M}(BMOA) = H^\infty \cap BMOA_{\log}$, where the logarithmic-$BMOA$ space $BMOA_{\log}$ consists of those $f \in \mathcal{H}ol(\mathbb{D})$ for which the Borel measure μ_f in $\mathbb{D}$ defined by $d\mu_f(z) = (1-|z|^2)|f'(z)|^2\,dA(z)$ is a 2-logarithmic Carleson measure.

Our starting point is the fact that whenever $p \neq q$, the only multiplier from $\mathcal{D}^p_{p-1}$ to $\mathcal{D}^q_{q-1}$ is the trivial one. It is easy to see that if $0 < p < q < \infty$ then $\mathcal{B} \cap \mathcal{D}^p_{p-1} \subset \mathcal{B} \cap \mathcal{D}^q_{q-1}$. This clearly implies the following: "*If X is a subspace of the Bloch space and $0 < p < q < \infty$, then the space of multipliers $\mathcal{M}(X\cap\mathcal{D}^p_{p-1}, X\cap\mathcal{D}^q_{q-1})$ is non trivial*". Then the question of characterizing the space $\mathcal{M}(X \cap \mathcal{D}^p_{p-1}, X \cap \mathcal{D}^q_{q-1})$ for classical subspaces of the Bloch space such as H^∞, $BMOA$ or $\mathcal{B}$ arises naturally. In this talk we shall consider the case $X = BMOA$. We shall present a number of results on the space $BMOA_{\log}$ and we shall use them to study the spaces $\mathcal{M}(BMOA\cap\mathcal{D}^p_{p-1}, BMOA\cap\mathcal{D}^q_{q-1})$, $0 < p, q < \infty$.

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