De Branges-Rovnyak spaces and generalized Dirichlet spaces

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The spaces now called de Brange-Rovnyak spaces were introduced by de Branges and Rovnyak in 1966. De Branges-Rovnyak spaces are subspaces of H^2 the standard Hardy space of the open unit disk $\mathbb{D}$. To give their definition we denote by $T_\chi,\ \chi \in L^\infty(\mathbb{T})$, where $\mathbb{T} = \partial\mathbb{D}$, the bounded Toeplitz operator on H^2, that is, $T_\chi f = P(\chi f)$, where P is the orthogonal projection of $L^2(\mathbb{T})$ onto H^2. Given a function b in the unit ball of H^∞, the *de Branges-Rovnyak space* $\mathcal{H}(b)$ is the image of H^2 under the operator $(I - T_b T_{\overline{b}})^{1/2}$. The space $\mathcal{H}(b)$ is given the Hilbert space structure that makes the operator $(I - T_b T_{\overline{b}})^{1/2}$ a coisometry of H^2 onto $\mathcal{H}(b)$, namely

$$\langle (I - T_b T_{\overline{b}})^{1/2} f, (I - T_b T_{\overline{b}})^{1/2} g\rangle_b = \langle f, g\rangle_2 \quad (f, g \in (\ker(I - T_b T_{\overline{b}})^{1/2})^\perp).$$

It turns out that if b is an inner function, then $\mathcal{H}(b) = (bH^2)^\perp$. Here we deal with the case when b is not an extreme point of the unit ball of H^∞. We describe the structure of some spaces $\mathcal{H}(b)$ and their connections with the generalized Dirichlet spaces defined below.

For $\lambda \in \mathbb{T}$ we define the local Dirichlet integral of f at λ by

$$D_\lambda(f) = \frac{1}{2\pi}\int_0^{2\pi} \left| \frac{f(\lambda) - f(e^{it})}{\lambda - e^{it}} \right|^2 dt.$$

where $f(\lambda)$ is the nontangential limit of f at λ. If $f(\lambda)$ does not exist, then we set $D_\lambda(f) = \infty$. Let μ be a positive Borel measure on $\mathbb{T}$. *The generalized Dirichlet space* $\mathcal{D}(\mu)$ consists of those functions $f \in H^2$ for which

$$D_\mu(f) = \int_{\mathbb{T}} D_\lambda(f) d\mu(\lambda) < \infty.$$

In 1997 D. Sarason showed that $\mathcal{D}(\delta_\lambda)$, where δ_λ is the unit mass at λ, can be identified with $\mathcal{H}(b_\lambda)$, where $b_\lambda(z) = (1 - w_0)\overline{\lambda} z/(1 - w_0\overline{\lambda} z)$, and $w_0 = (3 - \sqrt{5})/2$. Further results showing connection between the spaces $\mathcal{H}(b)$ and $D(\mu)$ have been recently obtained by T. Ransford, D. Guillot, N. Chevrot and C. Costara.