Besov regularity of stochastic partial differential equations on bounded Lipschitz domains

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The numerical methods for *Stochastic* PDEs studied so far in the literature are predominantly based on *uniform* space time refinements. However, in the *deterministic* world, there has been already shown that *non-uniform* refinement strategies like *adaptive* wavelet-based methods for elliptic PDEs on bounded Lipschitz domains $\mathcal{O} \subset \mathbb{R}^d$ can yield better convergence rates than uniform alternatives. Thus, it is an immediate question whether adaptivity pays also when dealing with Stochastic PDEs. In order to clarify this question one first of all needs to analyse the regularity of the solutions using the so-called *adaptivity scale*

$$B^{\alpha}_{\tau,\tau}(\mathcal{O}), \quad \frac{1}{\tau} = \frac{\alpha}{d} + \frac{1}{p}, \tag{1}$$

of Besov spaces to measure the smoothness with respect to the space variable ($p \geq 2$ is fixed). The regularity of a function in these Besov spaces determines the convergence rate of the best N-term wavelet approximation and therefore yields a benchmark for the convergence rate of adaptive wavelet methods.

In this talk, we first present recent results regarding the *spatial* regularity of the solution to stochastic parabolic second order PDEs on general bounded Lipschitz domains in the adaptivity scale (1) of Besov spaces. In particular, we will present a general embedding of weighted Sobolev spaces into Besov spaces. This is a key ingredient in order to obtain the desired results within the framework of the analytic approach initiated by N.V. Krylov.

Then we move on to the analysis of the joint *time-space* regularity of the solution. By a combination of the analytic approach with the semigroup approach of Da Prato/Zabczyk and its recent extension by van Neerven/Veraar/Weis we can establish an $L_q(L_p)$-regularity theory for SPDEs, where the integrability parameters q and p in time and space, respectively, might differ. As a consequence we can prove a time-space Hölder-Besov regularity result for the inhomogeneous stochastic heat equation with additive noise on general bounded Lipschitz domains.