The Hopf theorem for equivariant local maps

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Assume V is a real finite dimensional orthogonal representation of a compact Lie group G. Let Ω be an open invariant subset of V. We say that f is an *equivariant local map* if the domain of f is an open invariant subset of Ω, f is continuous equivariant and $f^{-1}(0)$ is compact. The space of equivariant local maps will be denoted by $\mathcal{F}_G(\Omega)$. Let $I = [0,1]$. We assume that the action of G on I is trivial. Let $\Lambda \subset I \times \Omega$ be an open invariant subset. Any equivariant map $h\colon \Lambda \to V$ such that $h^{-1}(0)$ is compact is called an *otopy*. Of course, otopy gives an equivalence relation on $\mathcal{F}_G(\Omega)$. The set of otopy classes will be denoted by $\mathcal{F}_G[\Omega]$. Assume that H is a closed subgroup of G. Recall that (H) stands for a conjugacy class of H and $WH = NH/H$, where NH is a normalizer of H in G. Let

$$\begin{aligned} \Phi(G) &= \{(H) \mid H \text{ is a closed subgroup of } G\},\\ \Phi_0(G) &= \{(H) \in \Phi(G) \mid \dim WH = 0\},\\ Iso(\Omega) &= \{(H) \in \Phi(G) \mid \Omega_{(H)} \neq \emptyset\}. \end{aligned}$$

It is well-known that the set $Iso(\Omega)$ is finite and so is $Iso(\Omega) \cap \Phi_0(G)$. We can now formulate our main result, which may be viewed as a local equivariant version of the well-known Hopf theorem.

Theorem. *There is a natural bijection*

$$\mathcal{F}_G[\Omega] \approx \prod_{(H)} \left(\sum_{i=1}^{n(H)} \mathbb{Z} \right),$$

where the product is taken over the set $Iso(\Omega) \cap \Phi_0(G)$ *and each direct sum is indexed by the set of connected components of the quotient* Ω_H/H.