Finite spaces and an axiomatization of the Lefschetz number

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In 2004 Arkowitz and Brown have presented an axiomatization of the reduced Lefschetz number of self-maps of finite CW-complexes. By a result of McCord, finite simplicial complexes are closely related to finite T_0-spaces. Using this connection, we show that the reduced Lefschetz number can be generalize to not only self-maps of finite spaces. Following May, we write $X^{(n)}$ for the nth barycentric subdivision of a finite T_0-space X. The main result states that the reduced Lefschetz number is the unique function λ on the class of maps $X^{(n)} \to X$ into integers satysfying the following conditions:

1. if $f\colon X^{(n)} \to X, g\colon X^{(m)} \to X$ are contiguous then $\lambda(f) = \lambda(g)$;
2. if $A \subseteq X$ and the following diagram

$$\begin{array}{ccccc} A^{(n+1)} & \longrightarrow & X^{(n+1)} & \longrightarrow & X^{(n+1)}/A^{(n+1)} \\ \downarrow \hat{f}' & & \downarrow f' & & \downarrow \overline{f}' \\ A' & \longrightarrow & X' & \longrightarrow & X'/A' \end{array}$$

   is commutative then $\lambda(f') = \lambda(\hat{f}') + \lambda(\overline{f}')$;
3. for any $f : X^{(n)} \to Y$ and $g : Y^{(m)} \to X$ we have $\lambda(gf^{(m)}) = \lambda(fg^{(n)})$;
4. let $f : \bigvee_{i=1}^{n} \mathcal{S}^{1,k} \to \bigvee_{i=1}^{n} \mathcal{S}^{1,2}$ for $k \geq 2$, then

$$\lambda(f) = -(\deg f_1 + \cdots + \deg f_n),$$

   where $f_i = p_i f e_i : \mathcal{S}_i^{1,k} \to \mathcal{S}^{1,2}$ for $i = 1, \ldots, k$ and $\mathcal{S}^{1,k}$ are combinatorial models of the circle $\mathbb{S}^1$.