Applications of fixed point theorems in equilibrium problems

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Let X be a nonempty set and $f : X \times X \to \mathbb{R}$ a real function such that $f(x, x) = 0$, for all $x \in X$. The classical equilibrium problem (abbreviated, EP) consists of

$$\text{finding } \tilde{x} \in X \text{ such that } f(\tilde{x}, x) \geq 0 \text{ for every } x \in X.$$

It is well known that equilibrium problems have many applications in optimization problems, Nash equilibrium problems, fixed point problems and variational inequalities problems. But, in this work, we will consider weaker equilibrium concepts. If (X, d) is a metric space, we say that $\tilde{x}$ is a local equilibrium for f if there is $\varepsilon > 0$ such that $f(\tilde{x}, x) \geq 0$ for all $x \in X$ which $d(x, \tilde{x}) < \varepsilon$. And, we say that $\tilde{x}$ is a weak local equilibrium for f if for all $\varepsilon > 0$ there exists $\delta > 0$ such that $f(\tilde{x}, x) \geq -\varepsilon d(x, \tilde{x})$ whenever $d(\tilde{x}, x) < \delta$.

Our main goal is to show the existence of the weak local equilibrium via the Lefschetz fixed point theorem for admissible multivalued mappings.

References

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