The spaces of proper and local maps are not homotopy equivalent

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This is a joint work with Piotr Bartłomiejczyk.

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Let $\mathcal{M}(X, Y)$ be the set of all continuous maps $f\colon D_f \to Y$ such that D_f is an open subset of X. Let us define the set of local maps

$$\mathcal{F}(n,k) = \{f \in \mathcal{M}(\mathbb{R}^{n+k}, \mathbb{R}^n) \mid f^{-1}(0) \text{ is compact}\}$$

and the set of proper maps

$$\mathcal{P}(n,k) = \{f \in \mathcal{M}(\mathbb{R}^{n+k}, \mathbb{R}^n) \mid f^{-1}(K) \text{ is compact for any compact set } K\}.$$

In [1] we introduce the topology on the set of local maps in more general setting and prove that the inclusion $\mathcal{P}(n,k) \subset \mathcal{F}(n,k)$ is a weak homotopy equivalence.

We denote by $\mathcal{F}_0(n,k)$ (resp. $\mathcal{P}_0(n,k)$) that component of $\mathcal{F}(n,k)$ (resp. $\mathcal{P}(n,k)$) which contains the empty map. In the talk we will present an essential complement to the above result. Namely, we will show that the spaces $\mathcal{F}_0(n,k)$ and $\mathcal{P}_0(n,k)$ are not homotopy equivalent for $n > 1$. Unfortunately, the problem in the case $n = 1$ remains unsolved.

References

[1] P. Bartłomiejczyk, P. Nowak-Przygodzki, *The exponential law for partial, local and proper maps and its application to otopy theory*, to appear in Comm. Contemp. Math.

[2] P. Bartłomiejczyk, P. Nowak-Przygodzki, *On the homotopy equivalence of the spaces of proper and local maps*, Cent. Eur. J. Math. 12(9) 2014, 1330–1336.