The family index theorem and bifurcation of solutions of nonlinear elliptic boundary value problems

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Session: 35. Topological fixed point theory and related topics

I will discuss some new criteria for bifurcation of solutions of nonlinear boundary value problems for elliptic systems. These criteria are derived explicitly from the coefficients of the leading terms of the linearization at the trivial branch in contradistinction with those obtained via the classical Lyapunov-Schmidt method which strongly depend on the lover order terms.

To this aim, I will introduce an "index of bifurcation" for parametrized families of nonlinear Fredholm maps whose non vanishing entails bifurcation from the trivial branch. In the case of elliptic BVP parametrized by $\mathbf{R}^k$ the "total index" can be computed from the Atiyah-Singer family index theorem and well known results about the J-homomorphism via a couple of reductions due to Agranovich and Dynin.

Time permitting, I will discuss an extension of the above result to more general parameter spaces using stable characteristic classes for spherical fibrations.

References

[1] J. Pejsachowicz, *The family index theorem and bifurcation of solutions of nonlinear elliptic BVP.* Journal of Differential Equations 252, 2012, 4942–4961.