Topological aspects of vector analogues of the Kalton-Roberts theorem

Tomasz Kochanek

Institute of Mathematics, Polish Academy of Sciences, Poland
tkoch@impan.pl

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We will discuss some topological aspects of the problem which asks to what extent the vector analogue of the Kalton-Roberts on nearly additive set functions holds true. Namely, we say that a Banach space X has the SVM (stability of vector measures) property (κ-SVM), provided there is a constant $v(X) < \infty$ such that for every set algebra $\mathcal{F}$ (with cardinality less than κ) and every function $\nu\colon \mathcal{F} \to X$ satisfying

$$\|\nu(A \cup B) - \nu(A) - \nu(B)\| \leq 1 \text{ for all } A, B \in \mathcal{F} \text{ with } A \cap B = \emptyset,$$

there exists a (finitely additive) vector measure $\mu\colon \mathcal{F} \to X$ satisfying $\|\mu(A) - \nu(A)\| \leq v(X)$ for each $A \in \mathcal{F}$. We will show, e.g., that for compact Hausdorff spaces K of finite Cantor-Bendixson height, the Banach space $C(K)$ has the ω_1-SVM property and in some cases ω_1 cannot be improved here. We will also show how some topological constructions may be used in order to prove that the Johnson-Lindenstrauss space with index $p = \infty$, JL_∞, has the ω_2-SVM property and, again, ω_2 is the best possible. Several open questions will be also addressed.